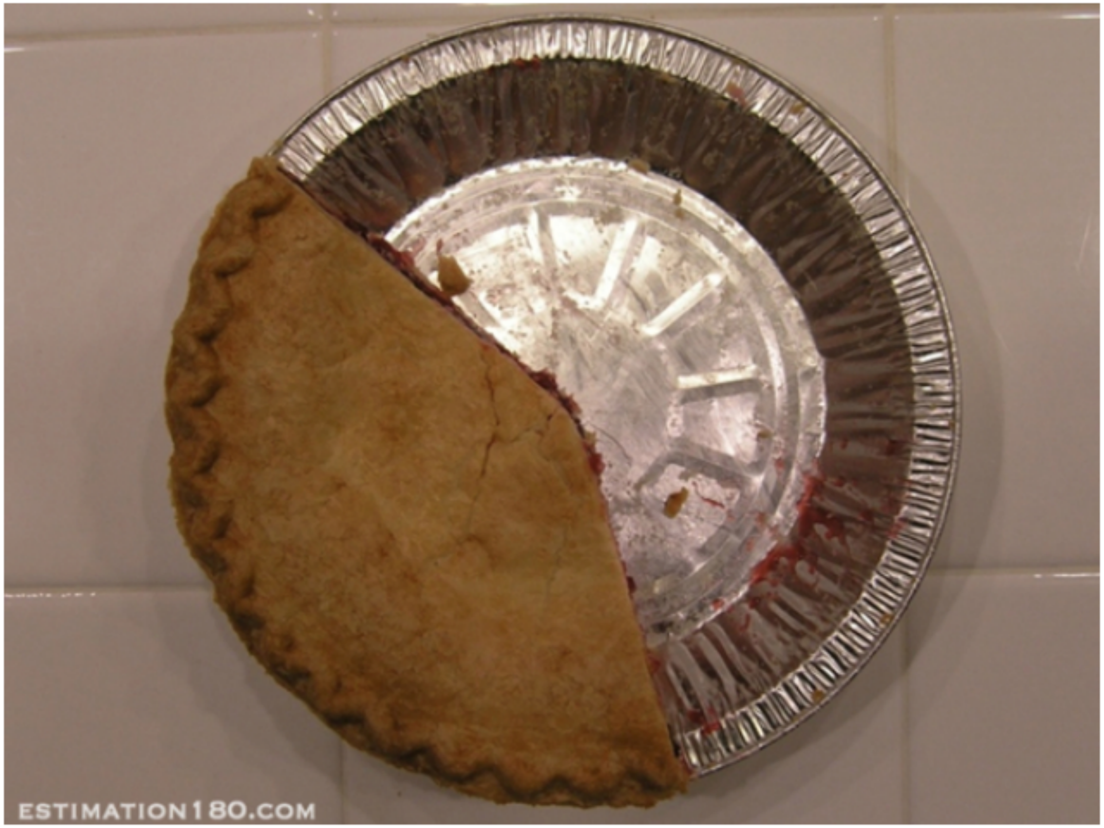
Created by Mr. Lischwe

WARMUP $12/\left(\sqrt{\sqrt{\sqrt{256}}}\right)$



What percent of the pie has been eaten?

http://www.estimation180.com/day-113.html

NEW UNIT: SEQUENCES

OBJECTIVES:

- Learn about what a sequence is
- Know the two most common types of sequences
- Write a <u>recursive rule</u> for a sequence

- 7, 9, 11, 13, 15, …

- This is an example of a sequence.
- How many terms do you see?
- How many terms could there be?
- Which term is in the third position?
- What would the term in the 6th position be?

SEQUENCES AND FUNCTIONS

- A ***sequence*** is a list of numbers in a specific order.
- Each element in a sequence is called a ***term***
- Each term has a ***position number***

term in the fourth position

7, 9, 11, 13, 15, 17, 19, 21, 23

term in the first position

DIFFERENCES BETWEEN SEQUENCES AND REGULAR FUNCTIONS

- A sequence has **NO ZEROTH TERM.** A sequence starts with the first term.
  - This is different than functions, when we usually think of the "original value" as the value when x is 0.

- There are **no decimal** term positions. You have the 1st term, 2nd term, 3rd term, with nothing in between
  - With functions, the "input" can be anything, including decimals

- If you were to graph a sequence (we usually don't), you would **NEVER** connect the points

CAN YOU FIND THE NEXT 3 TERMS?

1. 8, 15, 22, 29, … — 36, 43, 50 (always adding 7)
2. 10, 20, 40, 80, … — 160, 320, 640 (always multiplying by 2)
3. 5, 6, 8, 11, 15, … — 20, 26, 33 (adding 1,then 2, then 3, etc.)
4. 5.4, 4.2, 3, 1.8, … — 0.6, -0.6, -1.8 (always subtracting 1.2, or adding –1.2)
5. $0, \frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, \ldots$ — 3, 3¾, 4½, (always adding ¾)
6. $10, 5, \frac{5}{2}, \ldots$ — 5/4, 5/8, 5/16 (always dividing by 2, or multiplying by ½)

#1, #4, and #5 are called <u>arithmetic sequences</u>

#2, and #6 are called <u>geometric sequences</u>

#3 is neither one

2 MOST COMMON TYPES OF SEQUENCES

- **Arithmetic Sequence:** When the terms in the sequence have a **common difference (d)**
  - (Basically, a sequence that is linear)
- **Geometric Sequence:** When the terms in the sequence have a **common ratio (r)**
  - (Basically, a sequence that is exponential)
- BY THE WAY: "Arithmetic" as a noun is pronounced differently than "Arithmetic" as an adjective!

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

9, 13, 17, 21,…

Arithmetic
common difference: 4
next three terms: 25, 29, 33

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

10, 8, 5, 1,…

Neither; no common difference or ratio

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

7, 70, 700, 7000, …

Geometric

Common ratio = 10

Next 3 terms: 70000[redacted], 700000[redacted], 7000000[redacted]

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

8, 2, -4, -10…

Arithmetic
common difference: -6
next three terms: -16, -22, -28

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

–4, –2, 1, 5, …

Neither, no common difference or ratio

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

320, 80, 20, …

Geometric
Common ratio = ¼
Next 3 terms: 5, 5/4, 5/16

Determine whether the sequence appears to be an **arithmetic, geometric, or neither.** If arithmetic or geometric, find the next 3 terms, and write the common difference or common ratio.

$$-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \ldots$$

Arithmetic
common difference: 2/4 or 1/2
next three terms: 5/4 , 7/4, 9/4

Determine whether the sequence appears to be an **arithmetic sequence.** If so, find the common difference and the next three terms.

$$\frac{2}{3}, \frac{1}{3}, -\frac{1}{3}, -\frac{2}{3}, \ldots$$

Neither, no common difference between terms

I AM THINKING OF A SEQUENCE…

- The first term is 8.
- Can you tell me the sequence?

I AM THINKING OF A SEQUENCE…

- With each term, I am adding 4.
- Can you tell me the sequence?

I AM THINKING OF A SEQUENCE…

- The first term is 13.1 multiply the previous term by 2 to get the next term.
- Can you tell me the sequence?

RECURSIVE RULES

- We can precisely describe any sequence by stating the first term and describing how to get from one term to the next. This is called a **recursive rule.**

RECURSIVE RULES

- **10, 16, 22, 28, …**

- **FIRST TERM = 10**
- **ANY TERM = PREVIOUS TERM + 6**

RECURSIVE RULES

- **40, 60, 90, 135, …**

- **FIRST TERM = 40**
- **ANY TERM = PREVIOUS TERM x 1.5**

SEQUENCE NOTATION

- Things like "first term" and "previous term" are too long and wordy for mathematicians. Instead, we have a special notation for sequences, which is the letter "a" with a subscript:

- **a_1 = 1st term**
- **a_{13} = 13th term**
- **a_n = nth term**

DISCUSS: If "a_n" describes any term, how could you write "the term <u>before</u> a_n"???

RECURSIVE RULES

- **12, 20, 28, …**
- **FIRST TERM = 12**
- **ANY TERM = PREVIOUS TERM + 8**

- $a_1 = 12$
- $a_n = a_{n-1} + 8$

Why do we need both parts of this?

YOU CAN ALSO DO IT THIS WAY…

- **10, 16, 22, 28, …**
- **FIRST TERM = 10**
- **NEXT TERM = CURRENT TERM + 6**

- $a_1 = 10$
- $a_{n+1} = a_n + 6$

Determine whether the sequence is arithmetic or geometric. Then write the recursive rule for the sequence.

15, 26, 37, 48, …

$a_1 = 15$

$a_n = a_{n-1} + 11$

Determine whether the sequence is arithmetic or geometric. Then write the recursive rule for the sequence.

3, 12, 48, 192, …

$a_1 = 3;$

$a_n = 4 \cdot a_{n-1}$

ALTERNATE NOTATION FOR SEQUENCES…

- Although **subscript** notation is the most common way to write sequences, you can also use function notation.

- a_n **can also be written as** $f(n)$
- a_{n-1} **can also be written as** $f(n - 1)$
- a_{12} **can also be written as** $f(12)$
- **etc.**

Write the recursive rule for the sequence. <u>Use function notation!</u>

3, 23, 43, 63, …

$f(1) = 3;$

$f(n) = f(n - 1) + 20$

Write the recursive rule for the sequence. <u>Use function notation.</u>

6, 12, 24, 48, …

$f(1) = 6;$

$f(n) = 2 \cdot f(n - 1)$

Write the recursive rule for the sequence.

1/2, 1/8, 1/32, 1/128, …

$f(1) = \frac{1}{2};$

$f(n) = \frac{1}{4} \bullet f(n - 1)$

WHAT ARE THE FIRST FOUR TERMS OF THE SEQUENCE DEFINED BY THE RECURSIVE RULE?

$$a_1 = 4$$

$$a_n = a_{n-1} + 5$$

4, 9, 14, 19

WHAT ARE THE FIRST FOUR TERMS OF THE SEQUENCE DEFINED BY THE RECURSIVE RULE?

$$a_1 = 4$$

$$a_n = 5 \cdot a_{n-1}$$

4, 20, 100, 500

WHAT ARE THE FIRST FOUR TERMS OF THE SEQUENCE DEFINED BY THE RECURSIVE RULE?

$$a_1 = 4$$

$$a_{n+1} = a_n + 8$$

4, 12, 20, 28

WHAT ARE THE FIRST FOUR TERMS OF THE SEQUEN DEFINED BY THE RECURSIVE RULE?

$$a_1 = 4$$

$$a_{n+1} = 3 \cdot a_n$$

4, 12, 36, 108