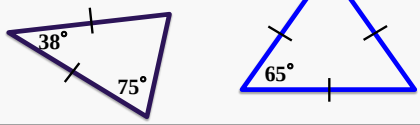
Created by Payton Nashe Warmup 3/

$(|8 \cdot \#of\ fingers\ on\ one\ hand| \div 40 \cdot \sqrt[3]{8} - 1)$

1) **Discussion** Consider the vertex and base angles of an isosceles triangle. Can they be right angles? Can they be obtuse? Explain.

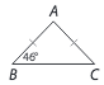
The vertex angle of an isosceles triangle can be acute, right, or obtuse as long as its measure is less than 180°. The base angles of an isosceles triangle can only be acute, meaning they have a measurement less than 90°. because otherwise they would cause the sum of the base angles to be ≥ 180° before adding in the third angle, which contradicts the Triangle Sum Theorem.

2) What's Wrong With Each Picture?



Homework
pg. 1104-1108 (4-9, 12, 13, 19, 20)

4. $m\angle A$

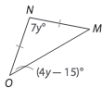


$m\angle A = 88°$

5. $m\angle R$

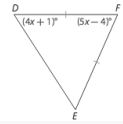
$m\angle R = (3x+1)° = (3(25)+1)° = (75+1)° = 76°$

6. $m\angle O$



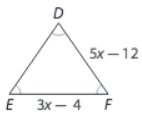
$m\angle O = (4y-15)° = (4(14)-15)° = (56-15)° = 41°$

7. $m\angle E$



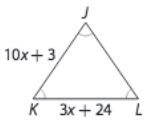
$m\angle E = (4x+1)° = (4(14)+1)° = (56+1)° = 57°$

8. $\overline{DE}$



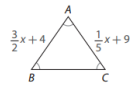
$DE = 8$

9. $\overline{KL}$



$KL = 33$

10. $\overline{AB}$



$\overline{AB} \cong \overline{AC}$, so $AB = AC$.

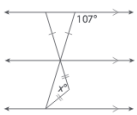
$$\frac{3}{2}x + 4 = \frac{1}{5}x + 9$$
$$\frac{13}{10}x + 4 = 9$$
$$\frac{13}{10}x = 5$$
$$x = \frac{50}{13}$$

$$AB = \frac{3}{2}\left(\frac{50}{13}\right) + 4$$
$$= \frac{75}{13} + \frac{52}{13}$$
$$= \frac{127}{13}$$
$$AB = \frac{127}{13}$$

12. Given $\triangle JKL$ with $m\angle J = 63°$ and $m\angle L = 54°$, is the triangle an acute, isosceles, obtuse, or right triangle?

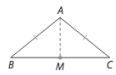
By the Triangle Sum Theorem, $m\angle K = 63°$, so the triangle is an acute isosceles triangle because all angle measures are less than 90°.

13. Find x. Explain your reasoning. The horizontal lines are parallel.



By the def. supp. ∠, the base angles of the top triangle have a measure of 73°. Therefore, the measure of the vertex angle is 34° by the Triangle Sum Theorem. The base angles of the bottom isosceles triangle will also measure 34° by the Vertical Angles Theorem. Thus, $x°$ will equal 112° by the Triangle Sum Theorem.

19. Critical Thinking Prove $\angle B \cong \angle C$, given point M is the midpoint of $\overline{BC}$.



Statements	Reasons
1. M is the midpoint of $\overline{BC}$.	1. Given
2. $\overline{BM} \cong \overline{CM}$	2. Definition of midpoint
3. $\overline{AB} \cong \overline{AC}$	3. Given
4. $\overline{AM} \cong \overline{AM}$	4. Reflexive Property of Congruence
5. $\triangle AMB \cong \triangle AMC$	5. SSS Triangle Congruence Theorem
6. $\angle B \cong \angle C$	6. CPCTC

20. Given that $\triangle ABC$ is an isosceles triangle and $\overline{AD}$ and $\overline{CD}$ are angle bisectors, what is $m\angle ADC$?

$m\angle BAC = m\angle BCA = 70°$, so $m\angle DAC = m\angle DCA = 35°$. Then, $m\angle ADC = 180° - (35° + 35°) = 110°$.

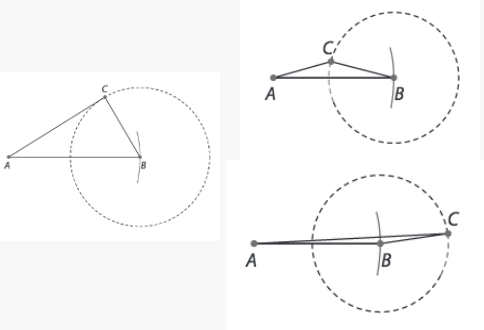
Objective

Explore Triangle Inequalities

Exploring possible triangles…

- **As we look at the various triangles, try to figure out the pattern of when it works and when it doesn't!**
- https://www.desmos.com/calculator/ym12g0rfjo

Another Way to Think about it…

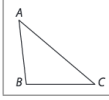


Triangle Inequality Theorem

pg. 1112

Triangle Inequality Theorem

The sum of any two side lengths of a triangle is greater than the third side length.



$AB + BC > AC$

$BC + AC > AB$

$AC + AB > BC$

To be able to form a triangle, each of the three inequalities must be true. So, given three side lengths, you can test to determine if they can be used as segments to form a triangle. To show that three lengths cannot be the side lengths of a triangle, you only need to show that one of the three triangle inequalities is false.

Tell whether a triangle can have sides with the given lengths. Explain.

7, 10, 19

$7 + 10 \stackrel{?}{>} 19$

$17 \not> 19$

No—by the Triangle Inequality Theorem, a triangle cannot have these side lengths.

Tell whether a triangle can have sides with the given lengths. Explain.

2.3, 3.1, 4.6

$2.3 + 3.1 \stackrel{?}{>} 4.6$ $\quad 5.4 > 4.6$ ✓

$2.3 + 4.6 \stackrel{?}{>} 3.1$ $\quad 6.9 > 3.1$ ✓

$3.1 + 4.6 \stackrel{?}{>} 2.3$ $\quad 7.7 > 2.3$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

Your Turn. pg. 1113 (6, 7)

6. 12 units, 4 units, 17 units

No; $12 + 4 \not> 17$

7. 24 cm, 8 cm, 30 cm

Yes; $24 + 8 > 30$, $8 + 30 > 24$, and $24 + 30 > 8$

The lengths of two sides of a triangle are 8 inches and 13 inches. Find the range of possible lengths for the third side.

$x + 8 > 13$ $\qquad$ $8 + 13 > x$

$x > 5$ $\qquad$ $21 > x$

Combine the inequalities. So $5 < x < 21$. The length of the third side is greater than 5 inches and less than 21 inches.

The lengths of two sides of a triangle are 22 inches and 17 inches. Find the range of possible lengths for the third side.

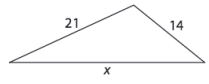
$x + 17 > 22$ $\qquad$ $22 + 17 > x$

$x > 5$ $\qquad$ $39 > x$

Combine the inequalities. So $5 < x < 39$. The length of the third side is greater than 5 inches and less than 39 inches.

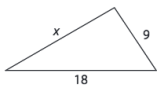
Your Turn! pg. 1115 (9, 10)

9.



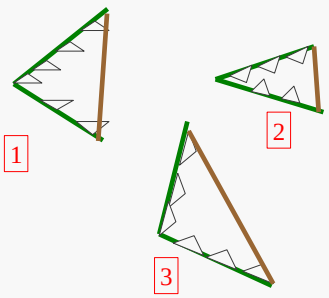
$7 < x < 35$

10.

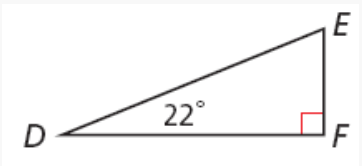


$9 < x < 27$

In Which Alligator's Mouth Can I fit the largest stick?

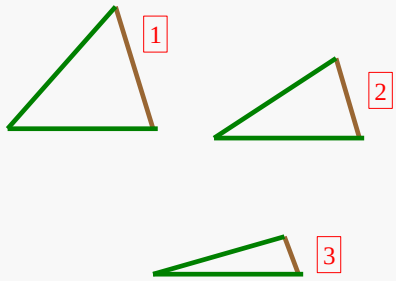


Which side of the triangle is the longest?

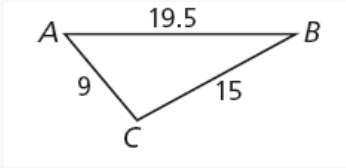


The largest angle is $\angle F$, so the longest side is $\overline{DE}$.

Which stick will have the largest alligator mouth?



Which angle of the triangle is the largest?



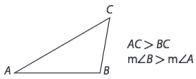
The longest side is $\overline{AB}$, so the largest angle is $\angle C$.

Side and Angle Relationships in Triangles

pg. 1115

Side-Angle Relationships in Triangles

If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.



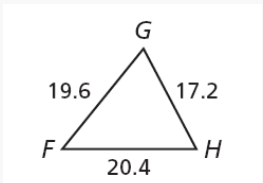
$AC > BC$
$m\angle B > m\angle A$

pg. 1116

Angle-Side Relationships in Triangles

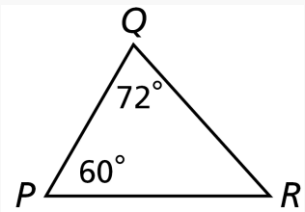
If two angles of a triangle are not congruent, then the longer side is opposite the larger angle.

Write the angles in order from smallest to largest.



The angles from smallest to largest are $\angle F$, $\angle H$ and $\angle G$.

Write the sides in order from shortest to longest.



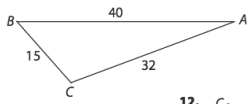
The sides from shortest to longest are $\overline{PQ}$, $\overline{QR}$, and $\overline{PR}$.

Your Turn!

pg. 1116 (11, 12)

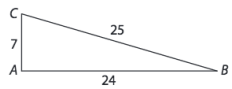
pg. 1117 (13, 14)

11.



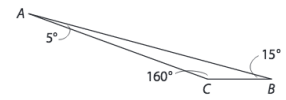
Longest side length: *AB*
Shortest side length: *CB*
$m\angle A$, $m\angle B$, $m\angle C$

12.



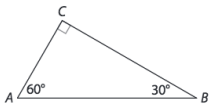
Longest side length: *BC*
Shortest side length: *AC*
$m\angle B$, $m\angle C$, $m\angle A$

13.



Greatest angle measure: m∠C
Least angle measure: m∠A
CB, AC, AB

14.



Greatest angle measure: m∠C
Least angle measure: m∠B
AC, BC, AB

Homework

Review Sheet